Mesa College – Math 104 (Trig) Challenge Exam SAMPLES

Directions: **NO CALCULATOR**. Write neatly and show your work and steps. Answers without appropriate work shown will receive little or NO credit**. Be sure to simplify all radicals and fractions.** Attach your neat and organized solution sheets behind this cover sheet. Make sure each solution is properly labeled.

#1 – 3. Use the given information to find the values of the remaining Trigonometric functions.

1. If $\sin\theta = -\frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$

2. If $\sin\theta = \frac{3}{5}$ and θ is in QII,

3. If $\cot\theta = -4$ and $270^o \le \theta < 360^o$

#4-6 . Find ALL values of x. Express your answers using radians.

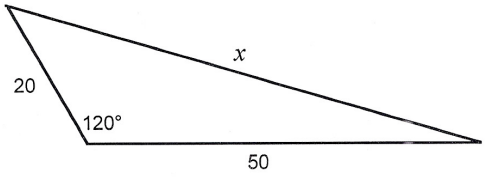
4. $3\csc x - 6 = 0.$

5. $\tan^2 x = -\tan x$

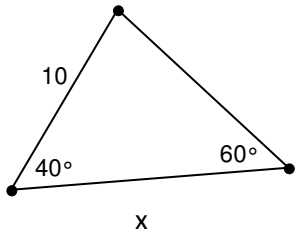
6. $2\sin^2 x - 5\sin x - 3 = 0$

7. Find the side length *x* in each of the triangles shown.

(7a)



(7b)



8. If $cos\,x = 0.8$ **and** $\sin x = -0.6$**, find :**

(8a) $\sin(2x)$ (8b) $\cos(2x)$ (8c) in which quadrant is angle (2x) ?

9. Given that $cos\,\alpha = \frac{2}{3}$ and $\sin\beta = -\frac{1}{4}$, where $\frac{-\pi}{2} \le \beta \le \frac{\pi}{2}$ and $0 \le \alpha \le \pi$ find :

(9a) $\sin(\alpha+\beta)$ (9b) $\cos(\alpha-\beta)$ (9c) $\cos\left(\frac{\alpha}{2}\right)$

(9d) in which quadrant is angle ($\alpha+\beta$)? (9e) in which quadrant is angle ($\alpha-\beta$)?

(9f) in which quadrant is angle $\left(\frac{\alpha}{2}\right)$?

10. Find the principal value of each:

10a) $Arc\cos\left(\frac{-\sqrt{3}}{2}\right)$ 10b) $\sin^{-1}\left(-\frac{1}{2}\right)$ 10c) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ 10d) $\operatorname{arc\,cot}\left(\sqrt{3}\right)$

10e) $\cos\left(\arcsin\left(\sqrt{3}/2\right)\right)$ 10f) $\cos\left(\arctan(-1)\right)$ 10g) $\cot^{-1}\left(\sin\frac{3\pi}{2}\right)$

11. Find the exact values of each:

11a) $\sin\left(\frac{\pi}{4}\right)$ 11b) $\cos\left(30^o\right)$ 11c) $\csc\left(\frac{-3\pi}{4}\right)$ 11d) $\sec\left(-420^o\right)$

11e) $\cot\left(\frac{13\pi}{6}\right)$ 11f) $\frac{\sin\left(\frac{\pi}{6}\right)}{1+\cos\left(\frac{\pi}{6}\right)}$ 11g) $\cos\left(45^o\right)\cos\left(60^o\right)-\sin\left(45^o\right)\sin\left(60^o\right)$

12. Simplify each expression completely.

(12a) $\sin(75^\circ)\cos(15^\circ) - \cos(75)\sin(15)$ (12b) $\sin^2\left(\frac{3\pi}{8}\right)-\cos^2\left(\frac{3\pi}{8}\right)$ (12c) $\cos\left(\frac{5\pi}{8}\right)$

13. Find all solutions between $0\le\theta<360^o$ for each:

(13a) $\cos^2\theta-2\sin\theta=-2$ (13b) $\csc\left(\frac{x}{2}\right)=\sqrt{2}$ (13c) $\sin(2x)=\cos(x)$

14. Sketch at least one period of the graph of each. Label the coordinates of one maximum point and the coordinates of one minimum point.

(14a) $f(x)=-2\cos\left(x+\frac{\pi}{2}\right)+1$ (14b) $g(x)=5\sin 2\left(x-\frac{\pi}{4}\right)$ (14c) $h(x)=3+2\sec x$

15. **Simplify the expression** $\frac{\cot y-1}{1-\tan y}$, so that it matches one of the expressions below. SHOW STEPS.

(a) $\cos y$ (b) $\tan y$ (c) 0 (d) $\frac{\sec y}{\csc y}$ (e) $\cot y$

16. Simplify each expression.

(16a) $(\sec\beta-\tan\beta)(\sec\beta+\tan\beta)$ (16b) $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}$

17. Complete the **trigonometric identity:** $\sec\theta-\cos\theta=w$, by selecting *w* from the list of expressions below.

(a) $\tan\theta\sin\theta$ (b) $\cot^2\theta\cos\theta$ (c) $\frac{1-\cos\theta\sin\theta}{\sin\theta}$ (d) $\frac{\sec\theta-1}{\sec\theta}$ (e) -1

18. SIMPLIFY each to a single trig expression involving no fractions.

(18a) $1+\frac{\tan^2 x}{1+\tan^2 x}$ (18b) $\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-1$

19. If $A = 3 + 2i$ and $B = 5 - i$, where $i = \sqrt{-1}$, find each:

(19a) A^2 (19b) AB (19c) $\frac{A}{B}$

20. Evaluate: $2i^2 - 5i^{75} + \frac{6}{i^{13}}$, where $i = \sqrt{-1}$.

21. Express each in standard form (i.e. $a \pm bi$)

(21a) $5\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) - i\sin\left(\frac{5\pi}{4}\right)\right)$ (21b) $8cis30^o$

22. Convert each from Polar coordinates to Cartesian coordinates

(22a) $\left(2, 30^o\right)$ (22b) $\left(-6, \frac{4\pi}{3}\right)$

23. Convert each from Cartesian coordinates to Polar coordinates

(23a) $\left(3, \sqrt{3}\right)$ (23b) $(-5, -5)$

24. If $4 + 3i \approx 5cis37^o$, find $\left(4 + 3i\right)^4$ in trigonometric form.

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Answers to Math 104 (Trig) SAMPLES

{ This is a DRAFT… If you find any errors, please contact me at: lafoster@sdccd.edu … Thanks. }

1. $\sin\theta = \frac{-1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, $\csc\theta = -2$, $\sec\theta = \frac{2\sqrt{3}}{3}$, $\tan\theta = \frac{-\sqrt{3}}{3}$, $\cot\theta = -\sqrt{3}$

2. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{-4}{5}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{-5}{4}$, $\tan\theta = \frac{-3}{4}$, $\cot\theta = \frac{-4}{3}$

3. $\sin\theta = \frac{-\sqrt{17}}{17}$, $\cos\theta = \frac{4\sqrt{17}}{17}$, $\csc\theta = -\sqrt{17}$, $\sec\theta = \frac{\sqrt{17}}{4}$, $\tan\theta = \frac{-1}{4}$, $\cot\theta = -4$

4. $\left\{x \mid x = \frac{\pi}{6} + 2k\pi \text{ or } x = \frac{5\pi}{6} + 2k\pi\right\}$ 5. $\left\{x \mid x = k\pi \text{ or } x = \frac{3\pi}{4} + k\pi\right\}$

6. $\left\{x \mid x = \frac{7\pi}{6} + 2k\pi \text{ or } x = \frac{11\pi}{6} + 2k\pi\right\}$ 7a. $x = 10\sqrt{39}$ 7b. $x = \frac{20\sin 80^o}{\sqrt{3}}$

8a. $\frac{-24}{25} = -0.96$ 8b. $\frac{7}{25} = 0.28$ 8c. Q-IV 9a. $\frac{5\sqrt{3}-2}{12}$ 9b. $\frac{2\sqrt{15}-\sqrt{5}}{12}$

9c. $\frac{+\sqrt{30}}{6}$ 9d. 9e. 9f. all are in Q-I 10a. $\frac{5\pi}{6}$ 10b. $\frac{-\pi}{6}$ 10c. $\frac{\pi}{6}$ 10d. $\frac{\pi}{6}$

10e. $\frac{1}{2}$ 10f. $\frac{\sqrt{2}}{2}$ 10g. undefined. 11a. $\frac{\sqrt{2}}{2}$ 11b. $\frac{\sqrt{3}}{2}$

11c. $-\sqrt{2}$ 11d. 2 11e. $\sqrt{3}$ 11f. $2-\sqrt{3}$ 11g. $\frac{\sqrt{2}-\sqrt{6}}{4}$

12a. $\frac{\sqrt{3}}{2}$ 12b. $\frac{\sqrt{2}}{2}$ 12c. $\frac{\sqrt{2-\sqrt{2}}}{2}$ 13a. $\{90^o\}$

13b. $\{90^o, 270^o\}$ 13c. $\{90^o, 270^o, 30^o, 150^o\}$ 14. GRAPHS… see below

15. E 16a. 1 16b. $2\sec\theta$ 17. A

18a. $\cos^2 x$ 18b. $\sin x$ 19a. $5+12i$

19b. $17+7i$ 19c. $\frac{1}{2}+\frac{1}{2}i$ 20. $-2-i$

21a. $-5+5i$ 21b. $4\sqrt{3}+4i$ 22a. $(\sqrt{3},1)$ 22b. $(3,3\sqrt{3})$

23a. $\left(2\sqrt{3}, \frac{\pi}{6}\right)$

23b. $\left(5\sqrt{2}, \frac{5\pi}{4}\right)$

24. $625cis(148^o)$