# Mesa College – Math 96 - Challenge Exam SAMPLES

Directions: **NO CALCULATOR**. Unless otherwise noted, you may assume that you are solving and evaluating over the set of Real Numbers. Write neatly and show your work and steps. Answers without appropriate work shown will receive NO credit**. Be sure to simplify all radicals and fractions.** Attach your neat and organized solution sheets behind this cover sheet. Make sure each solution is properly labeled.

1. Solve each: (a) $\begin{cases} 2x-9y=-31 \\ 3x+2y=0 \end{cases}$ (b) $\begin{cases} 2x^2-y=9 \\ y=-3x \end{cases}$

2. Graph each inequality. Label the coordinates of the x- and y- intercepts ON the graph:

   (a) $x-2y<1$ (b) $5x-3y>-10$ (c) $y>\frac{1}{2}x^2-2$

3. Solve each: (3a) $2x^2-x-6\le 0$ (3b) $\frac{2}{x-3}\ge 1$

4. Solve each: (a) $4|3x-1|-14>2$ (b) $5-3|4x+6|\le 32$ (c) $6+|2x^2-1|\le 2$

5. Solve each over the set of Complex numbers:

   (a) $x^2-6x+58=0$ (b) $3x^2-4x=-5$ (c) $\frac{4}{x}+\frac{1}{x+2}=\frac{3}{2}$

6. Solve: (6a) $\sqrt{y+41}=y-1$ ( 6b) $3(2x-5)^2-11(2x-5)=20$ (6c) $2x+2\sqrt{x}+1=0$

7. Solve each: (7a) $125^{x+4}=5^{5x-1}$ (7b) $81^{3x-9}=27^{7x-2}$ (7c) $2^{x+1}\bullet 4^{x+1}=8^{x+5}$

8. SOLVE: (a) $\log_3(x)-\log_3(x+6)=3$ (b) $\log_3 x+\log_3(x+6)=2$ (c) $\log_2(x+2)+\log_2(x-3)=3$

9. Given $f(x)=3x+2$ and $g(x)=x^2-x$, find the following:

   (a) $g(f(-3))$ (b) $(g\circ f)(x)$ (c) $(f\circ f)(x)$

10. Find the inverse of each function:

    (a) If $f(x)=\log_7(x-2)$, find $f^{-1}(x)$ (b) If $g(x)=5^{x+2}$, find $g^{-1}(x)$

11. Find the linear equation of each line using the given information:

    (a) through the points (-2, 5) and (7, -6).

    (b) through the point (4, 6) and perpendicular to the line $3x+y=-8$

**#12-13. Find the EXACT value of the indicated variable. Remember: NO CALCULATOR.**

12. Solve for 't' : $7 = e^{-0.4t}$

13. Solve for 'x' : $144 = 5^{2x+1}$

14. The population of a colony of bacteria is given by: $N(t) = 3000(2)^{t-1}$, where t represents time in minutes.

(a) How many bacteria are present initially? (b) How many bacteria are present after 3 minutes?

15. For the arithmetic sequence {9, 16, 23, 30, …. }

(a) Find the 100th term. (b) Find the sum of the first 100 terms.

16. For the geometric sequence $\{8, 4, 2, 1, ...\}$ find the fractional value (i.e. no decimals) of each:

(a) the 32nd term (b) the sum of the first 8 terms.

17. Find the center and radius (or radii ) of the following:

(a) circle: $x^2 + y^2 + 8x - 6y - 24 = 0$ (b) ellipse: $9x^2 + 4y^2 + 54x - 16y + 61 = 0$

18. Simplify each completely. You may assume that no variable represents a negative Real value.

(a) $\dfrac{5\sqrt{6}}{2\sqrt{3}}$ (b) $\sqrt[4]{96x^9y^8z^{10}}$ (c) $5\sqrt{27} - 6\sqrt[3]{24} + 7\sqrt{3}$ (d) $\dfrac{\sqrt[3]{48x^6}}{\sqrt[3]{64x^{11}}}$

19. Simplify the following completely. The final answers should contain only positive exponents.

(a) $\dfrac{9a^{\frac{4}{3}}b^{-0.25}}{-12a^{\frac{-2}{3}}b^{\frac{7}{4}}}$ (b) $\left(\dfrac{-6a^3b^{-5}c^{-3}}{8a^{-3}b^{-7}c^7}\right)^{-3}$

20. Starting from the same point, a senior citizen drove north for 2 hours at a rate of 50 miles per hour and a biker drove east for one hour at 100 miles per hour. How far apart are they when they stop?

21. The diagonal of a rectangle divides the opposite angles into $30^o$ and $60^o$ angles. If the length of the diagonal is 12 inches, find the exact value of the perimeter of the rectangle.

22. Find the volume of a right circular cylinder with height 30 meters and radius of base 4 meters.

23. Find the volume of a sphere with radius 12 cm.

24. For each function, graph the function showing the coordinates of the vertex, and the x- and y-intercepts **on the graph**.

(a) $f(x) = x^2 - 2x - 15$ (b) $f(x) = 4x^2 + 16x + 15$ (c) $x = -2(y+3)^2 - 8$

**ANSWERS for the Mesa College - Math 96 Challenge Exam SAMPLES**

**Remember: No calculators of any form, and all answers should be completely simplified.**

**NOTE: The final solution to an <u>equation</u> belongs inside { } symbols [ see #5a or #6a ]. The solution to an <u>inequality</u> belongs in either { } symbols or interval notation [ see #3a ].**

1a) $\{\,(-2,\,3)\,\}$ 1b) $\left\{\,(-3,\,9),\ \left(3/2,\,-9/2\right)\,\right\}$

2) see last page

3a) $\{x \mid \frac{-3}{2} \le x \le 2\}$ *or* $[\frac{-3}{2}, 2]$ 3b) $\{x \mid 3 < x \le 5\}$ *or* $(3,5]$

4a) $\{x \mid x < -1 \text{ or } x > \frac{5}{3}\}$ *or* $(-\infty,-1) \cup (\frac{5}{3}, \infty)$ 4b) $\Re$ or 'identity' 4c) 'no solution' or { }

5a) $\{\,3 \pm 7i\,\}$ 5b) $\left\{\,\frac{2}{3} \pm \frac{i\sqrt{11}}{3}\,\right\}$ 5c) $\left\{\,\frac{2 \pm 2\sqrt{13}}{3}\,\right\}$

6a) y = 8, -5, but -5 rejects, so $\{8\}$ 6b) $\{\,\frac{11}{6},\,5\,\}$ 6c) $\left\{\,\frac{\pm 1}{2} i\,\right\}$

7a) $\left\{\,\frac{13}{2}\,\right\}$ 7b) $\left\{\frac{-10}{3}\right\}$ 7c) $\{\ \}$ *or* '*no solution*'

8a) $\left\{\frac{-81}{13}\right\}$ 8b) $\left\{-3 \pm 3\sqrt{2}\right\}$ 8c) $x = \frac{1 \pm \sqrt{57}}{2}$, but only one 'checks,' so: $\left\{\frac{1+\sqrt{57}}{2}\right\}$

9a) $g(f(-3)) = 56$ 9b) $(g \circ f)(x) = 9x^2 + 9x + 2$ 9c) $(f \circ f)(x) = 9x + 8$

10a) $f^{-1}(x) = 7^x + 2$ 10b) $g^{-1}(x) = \log_5(x) - 2$

11a) $y = \frac{-11}{9}x + \frac{23}{9}$ or 11x + 9y = 23 11b) $y = \frac{1}{3}x + \frac{14}{3}$ or x – 3y = -14

12) $\left\{\,\frac{\ln 7}{-0.4}\,\right\}$ 13) $\left\{\frac{\log_5(144)-1}{2}\right\}$ *or* $\left\{\frac{2\log_5(12)-1}{2}\right\}$

14a) Initially, there were 1500 bacteria. 14b) After 3 minutes there were 12,000 bacteria

15a) The 100th term is: 702 15b) The sum of the first 100 terms is: 35,550.

16a) The 32nd term is: $\frac{1}{2^{28}}$ 16b) The sum of the first 8 terms is: $\frac{255}{16}$

#17. Use "Completing the Square" on each to obtain:

17a) $(x + 4)^2 + (y - 3)^2 = 7^2$ This is a circle with center = (-4, 3) and radius = 7

17b) $\frac{(x+3)^2}{2^2} + \frac{(y-2)^2}{3^2} = 1$ This is an ellipse with center = (-3, 2)

and x-radius = $\pm 2$, and y-radius = $\pm 3$

Don't forget about the graphs of: hyperbolas, square roots, absolute value, exponential and logarithmic functions.

18a) $\frac{5\sqrt{2}}{2}$ 18b) $2x^2y^2z^2\sqrt[4]{6xz^2}$ 18c) $22\sqrt{3} - 12\sqrt[3]{3}$ 18d) $\frac{\sqrt[3]{6x}}{2x^2}$

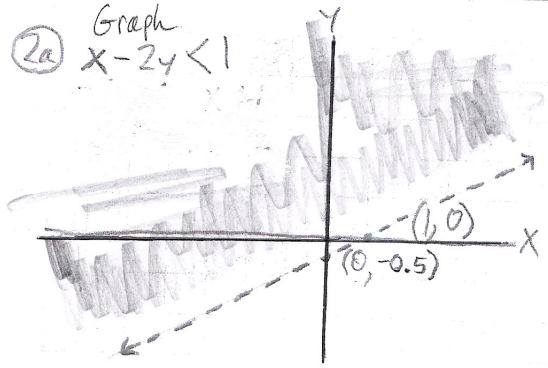
19a) $\frac{-3a^2}{4b^2}$ 19b) $\frac{-64c^{30}}{27a^{18}b^6}$

20. They will be $100\sqrt{2}$ miles apart when they stop. 21. The perimeter is $\left(12 + 12\sqrt{3}\right)$ inches.

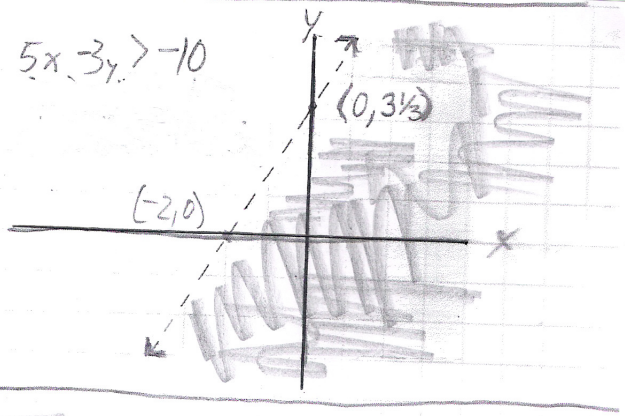
22. The volume is $480\pi\, m^3$ 23. The volume is $2304\pi\, cm^3$

24. see GRAPHS on the next page.

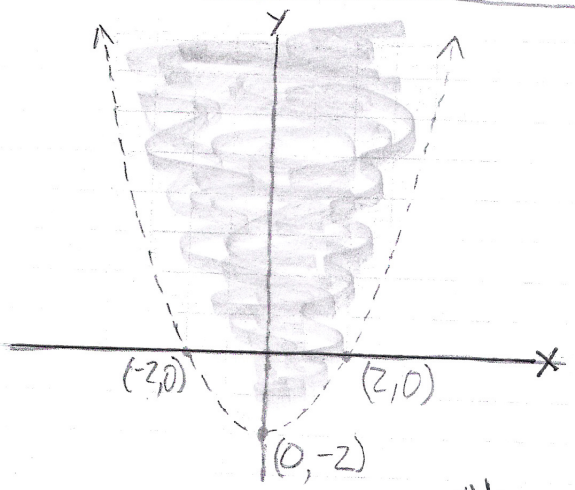
(2a) Graph $x - 2y < 1$



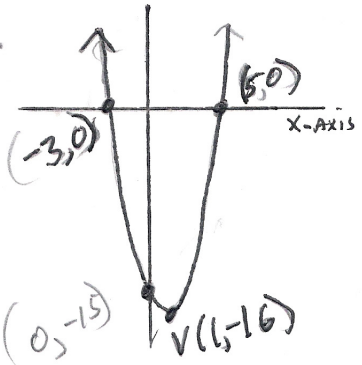

(2b) $5x - 3y > -10$




(2c) $y > \frac{1}{2}x^2 - 2$

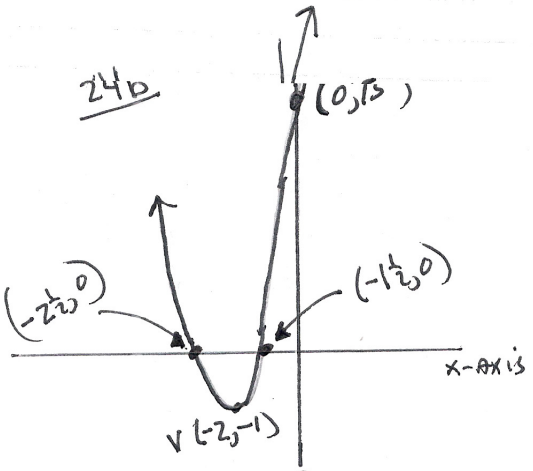



24a

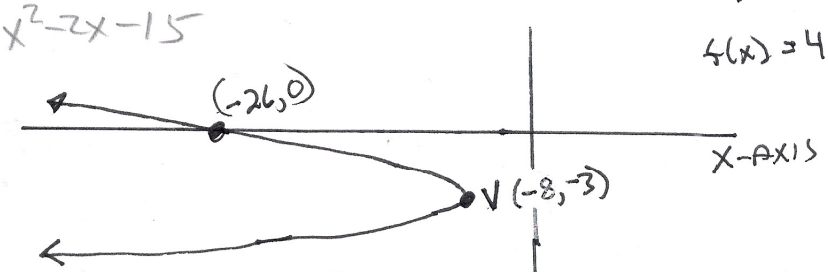



$f(x) = x^2 - 2x - 15$

24b




$f(x) = 4x^2 + 16x + 15$




24c $x = -2(y+3)^2 - 8$