Mesa College – Math 116 - SAMPLES

Directions: **NO CALCULATOR**. Write neatly, show your work and steps. Label your work so it's easy to follow. Answers without appropriate work will receive NO credit. **For final answers, be sure to simplify all radicals and fractions.**

#1. Find the domain of each function. Express your answer in <u>interval notation</u>.

1a) $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$ 1b) $g(x)=\dfrac{\sqrt{3-5x}}{4x^2-25}$ 1c) $h(x)=\dfrac{x^2-3x-10}{\sqrt{3x-8}}$

#2. Find $\dfrac{f(a+h)-f(a)}{h}$. Completely simplify your result, where:

2a) $f(x)=-x^2+x+5$ 2b) $f(x)=2x^2-5x$ 2c) $f(x)=\dfrac{2}{x}$

#3. Find $(f\circ g)(x)$, and specify the domain of $f\circ g$ using <u>interval notation,</u> where:

3a) $f(x)=\dfrac{x}{3x+2}$ and $g(x)=\dfrac{2}{x}$ 3b) $f(x)=\dfrac{2x}{x-4}$ *and* $g(x)=\dfrac{2}{x+1}$ 3c) $f(x)=g(x)=\dfrac{x}{x+1}$

#4. Find all x such that $f(x)\geq 0$. Write your answer using <u>interval notation</u>, where:

4a) $f(x)=-x^4+x^3+2x^2$. 4b) $f(x)=2x^3-x^2-8x+4$ 4c) $f(x)=\dfrac{2x-4}{x^2-9}$

#5. Find the equations of all vertical and horizontal asymptotes for :

5a) $f(x)=\dfrac{x^2-2x+1}{x^3-x^2+x-1}$ 5b) $g(x)=\dfrac{2x^2-x-1}{4x^2-x-3}$ 5c) $h(x)=\dfrac{5x^3+3x}{4x^3-9x}$

#6. Express as a single logarithm. Assume all values are properly defined

6a) $\log\dfrac{x^2}{y^3}+4\log y-6\log\sqrt{xy}$ 6b) $\log_b\dfrac{b}{\sqrt{x}}+\log_b\sqrt{bx}$

#7. Solve:

7a) $\log_3 3x=\log_3 x+\log_3(4-x)$. 7b) $\log_3(x)-\log_3(x+1)=3$ 7c) $\log_x 25=\dfrac{2}{3}$

7d) $\log_3 x+\log_3(x+6)=2$ 7e) $27^{2x-5}=9\left(3^{7x+1}\right)$ 7f) $\log_2(3x-1)=5$

7g) $23=e^{-0.4x}$ 7h) $\log_x 5=3$

#8. Suppose $A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$, $B=\begin{bmatrix}-1\\ 5\end{bmatrix}$, and $C=\begin{bmatrix}0 & 3\\ -2 & 1\end{bmatrix}$. Find the following:

8a) AB 8b) 3A - 2C 8c) CB 8d) BC 8e) A^{-1}

#9. Use augmented matrices and Gaussian elimination to solve:

9a) $\begin{cases}3x+y-2z=1\\ 2x+3y-z=2\\ x-2y+2z=-10\end{cases}$ 9b) $\begin{cases}x-2y+3z=9\\ -x+3y=-4\\ 2x-5y+5z=17\end{cases}$ 9c) $\begin{cases}2x+3y=-2\\ 5y-2z=4\\ 4x+3z=-7\end{cases}$

#10. Maximize $P=10x+7y$, given the constraints $0\le x\le 60,\ 0\le y\le 45,$ and $5x+6y\le 420$. Sketch the graph of the constraint inequalities. Label the critical vertices on the graph.

#11. An accountant prepares tax returns for individuals and for small businesses. On average, each individual return requires 3 hours of her time, and 1 hour of computer time. Each business requires 4 hours of her time and 2 hours of computer time. Because of other business considerations, her time is limited to 240 hours, and the computer time is limited to 100 hours. If she earns a profit of $80 per each individual return, and a profit of $150 on each business return, how many returns of each type should she prepare to maximize her profit?

#12. SOLVE each over the complex number system.

12a) $x^4-5x^2=36$ 12b) $x^{\frac{-4}{3}}-5x^{\frac{-2}{3}}+4=0$

12c) $\sqrt{2x^2-7}=x+3$ 12d) $2\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+5=0$

#13. Sketch the graph of each polynomial function labeling the zeros and the y-intercepts for each.

13a) $y=2x^3-5x^2-4x+3$ 13b) $y=x^4-3x^2+2x$

#14. Let F(x) be a polynomial function with rational coefficients. Solve each, over the complex numbers, if necessary, using the given hints.

14a) $x^3+2x^2-23x-60=0$, where 5 is a root.

14b) $2x^4-3x^3-7x^2-8x+6=0$, where $\frac{1}{2}$ is a root

#15a) Write a third degree polynomial equation with rational coefficients, that has $2\sqrt{5}$ and 1 as its roots

#15b) Write a fourth degree polynomial equation with integer coefficients that has $3i$ (where $i=\sqrt{-1}$) and -1 as roots, where -1 is a 'double' root. (also called: -1 has a multiplicity of two.)

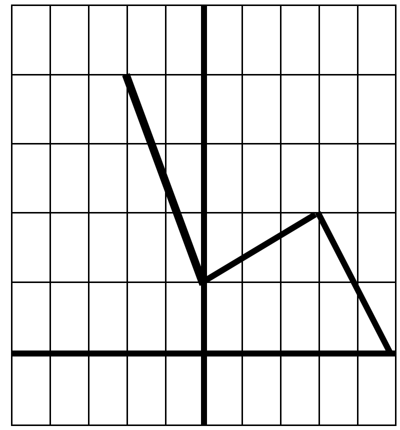
#16. The below graph represents part of a function y = f(x). Sketch each:

16a) y = f(x+2)   16b) y = $f(2x)$

16c) y = $f^{-1}(x)$ 16d) $y = f(x) - 2$

16e) the reflection of f(x) over the y-axis

16f) the reflection of f(x) over the x-axis



#17. Suppose f(x) is a continuous, one-to-one function, find:

17a) $f(-2)$, if $f(2) = 5$, and f(x) is an 'even' function

17b) $f(-2)$, if $f(2) = 5$, and f(x) is an 'odd' function

#18. Sketch the graph of each piece-wise function. Label the coordinates of each vertex and/or each point of discontinuity.

18a) $y = \begin{cases} |x|, \text{ if } -2 \le x < 1 \\ 2x - 1, \text{ if } x \ge 1 \end{cases}$ 18b) $y = \begin{cases} 1, \text{ if } x < -2 \\ x + 1, \text{ if } -2 \le x \le 3 \\ 2, \text{ if } x > 3 \end{cases}$

#19. Resolve each into its partial fractions.

19a) $\dfrac{x+2}{2x^2 - 7x - 15}$ 19b) $\dfrac{2x^2 + 7x + 23}{(x-1)(x+3)^2}$

#20. A new drug is injected into a patient. The number of milligrams remaining in the patient's bloodstream 't' hours is modeled by: $D(t) = 60e^{-0.2t}$. (a) How many milligrams were initially injected? (b) How many milligrams of the drug remain after 5 hours? (c) When will there be one-forth of the initial dosage in the patient's bloodstream?

#21. Students in a math class took a final exam. As part of a study, they were tested each month thereafter to determine how much they remembered. The following formula was derived: $S(t) = 77.4 - 20\log(t+1),\ t \ge 0$. Where S(t) represents the score (as a percent), and 't' represents months. (a) What was the average initial score on the exam? (b) What was the average score after 9 months?

ANSWERS to Math 116 Challenge Exam – SAMPLES

Disclaimer: A few of these answers may (will) be wrong! There's a myriad of reasons why: I'm stooped, my fingers are fat, I'm dyslexic, I'm tired, the summer sun has baked my brain. Please, email me your suggested solution(s), I'll double check it, and get back to you. Thanks. Larry Foster: lafoster@sdccd.edu

NOTE: I have NOT yet included the GRAPHS … someday.

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1a) Domain = $\left[\frac{3}{4},2\right)\cup\left(2,\infty\right)$ 1b) Domain = $\left(-\infty,\frac{-5}{2}\right)\cup\left(\frac{-5}{2},\frac{3}{5}\right]$ 1c) Domain = $\left(\frac{8}{3},\infty\right)$

2a) -2a + 1 – h 2b) 4a – 5 + 2h 2c) $\dfrac{-2}{a^2+ah}$

3a) $f\circ g=\dfrac{1}{x+3}$, domain = $(-\infty,-3)\cup(-3,0)\cup(0,\infty)$

3b) $f\circ g=\dfrac{-2}{2x+1}$, domain = $(-\infty,-1)\cup\left(-1,\frac{-1}{2}\right)\cup\left(\frac{-1}{2},\infty\right)$

3c) $f\circ g=\dfrac{x}{2x+1}$, domain = $(-\infty,-1)\cup\left(-1,\frac{-1}{2}\right)\cup\left(\frac{-1}{2},\infty\right)$

4a) $f(x)>0, when\ x\in\left[-1,2\right]$ 4b) $f(x)>0, when\ x\in\left[-2,\frac{1}{2}\right]\cup\left[2,\infty\right)$

4c) $f(x)>0, when\ x\in\left[-3,2\right]\cup\left(3,\infty\right)$

5a) $f(x)=\dfrac{x-1}{x^2+1}, x\neq 1$, vertical Asymptote: none, horizontal Asymptote: $y=0$,
Point of Discontinuity (aka: 'hole'): $(0,-1)$

5b) $g(x)=\dfrac{2x+1}{4x+3}, x\neq 1$, vert: $x=\frac{-3}{4}$, horiz: $y=\frac{1}{2}$, hole @ $\left(1,\frac{1}{3}\right)$

5c) $h(x)=\dfrac{5x+3}{(2x+3)(2x-3)}, x\neq 0$, vert: $x=\pm\frac{3}{2}$, horiz: $y=0$, hole @ $\left(0,\frac{-1}{3}\right)$

6a) $-\log(xy)$ 6b) $1\frac{1}{2}$

7a) $\{1\}$, 0 rejects 7b) $\{\ \}$, $\dfrac{-27}{26}$ rejects 7c) $\{125\}$ 7d) $\{3\}$, -9 rejects

7e) $\{-18\}$ 7f) $\{11\}$ 7g) $\left\{\dfrac{\ln 23}{-0.04}\right\}$ this is best possible w/o calculator 7h) $\left\{\sqrt[3]{5}\right\}$

8a) $\begin{bmatrix}-19\\-19\end{bmatrix}$ 8b) $\begin{bmatrix}3&0\\16&-11\end{bmatrix}$ 8c) $\begin{bmatrix}-10\\7\end{bmatrix}$ 8d) not defined 8e) $\begin{bmatrix}\frac{3}{11}&\frac{2}{11}\\\frac{4}{11}&\frac{-1}{11}\end{bmatrix}$

9) after a LOT of work: 9a) $\{(-2, 1, -3)\}$ 9b) $\{(1, -1, 2)\}$ 9c) $\{(-4, 2, 3)\}$

Remember: the parenthesis inside of the solution set is mandatory!

10) See below for sketch. corners: (0,0), (0,45), (60,0), 30,45), (60,20) The max value of P occurs when x = 60 and y = 20.

11) Let x = # of individual returns, y = # of business returns Objective function: P = 80x + 150y

Constraints: $\begin{cases} x \geq 0 \\ y \geq 0 \\ 3x + 4y \leq 240 \\ x + 2y \leq 100 \end{cases}$ corners: (0,0), (0, 50), (40,30), 80,0)

Max profit occurs @ (40,30) she should prepare 40 individual returns, and 30 business returns.

12a) $\{\pm 3, \pm 2i\}$ 12b) $\{\frac{1}{8}, 1\}$ 12c) $\{8, -2\}$ 12d) $\{2, \frac{1}{2}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\}$

#13 See below for sketches. Using a combination of: Rational Root theorem, synthetic division, factoring, and quadratic formula:

13a) $y = (x+1)(2x-1)(x-3)$ zeros: $-1, 3, \frac{1}{2}$, y-intercept: $(0, 3)$ + check out it's 'extreme' behavior

13b) $y = x(x-1)^2(x+2)$, zeros: 0, -2, 1 (multiplicity of 2), y-int: (0,0)

#14 – see 'hints' from #13

14a) (x – 5)(x + 3)(x + 4) = 0 $\Rightarrow \{5, -3, 4\}$

14b) $(x - \frac{1}{2})(x-3)(x^2 + 2x + 2) = 0 \Rightarrow \{\frac{1}{2}, 3, -1 \pm i\}$

15a) remember with rational coefficients, each irrational and complex root must include its conjugate.

$$(x - 2\sqrt{5})(x + 2\sqrt{5})(x-1) = 0 \Rightarrow x^3 - x^2 - 20x + 20 = 0$$

15b) $(x-3i)(x+3i)(x+1)(x+1) = 0 \Rightarrow x^4 + 2x^3 + 10x^2 + 18x + 1 = 0$

#16 graphs later. Each corner of the shape should be modified as indicated below. Then, connect the dots.

16a) shift entire graph left 2 units. (subtract 2 from x-values, keep y-values the same)

16b) (divide x-values by 2, keep y-values the same)

16c) swap all x$\leftrightarrow$y that is: (-2,4) $\rightarrow$ (4, -2)

(0,1) $\rightarrow$ (1,0)

(3,2) $\rightarrow$ (2,3)

(5,0) $\rightarrow$ (0,5)

16d) shift entire graph <u>down</u> 2 units: (keep x-value the same, subtract 2 from y-values)

16e) (change all x-values to their opposite, keep y-value the same)

16f) (keep x-value same, change y-values to their <u>opposite)</u>

17a) $f(-2)=5$ 17b) $f(-2)=-5$

#18 – see below for sketches (someday)

19a) $\frac{-1}{13(2x+3)}+\frac{7}{13(x-5)}$ 19b) $\frac{2}{x-1}+\frac{-5}{(x+3)^2}+\frac{0}{x+3}$ of course you don't need this last fraction

20a) 60 mg 20b) $60e^{-1}$ mg, or $\frac{60}{e}$ mg 20c) $\frac{\ln(\frac{1}{4})}{-0.2}$ hours, or $\frac{\ln(4)}{0.2}$ hours.

21a) The initial average score was 77.4% 21b) After 9 months, the resulting average score was 57.4%